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(6) THE CALIBRATION
OF
A LOW-FREQUENCY CALIBRATING SYSTEM

by

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U. S. NAVY UNDERWATER SOUND REFERENCE LABORATORY

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ABSTRACT

The receiving frequency responses of hydrophones in the 1- to 100-cps range are determined at the USRL in a closed tank system. The procedure requires that certain physical constants of the electrodynamic driver and the water-filled tank be known. The techniques for determining these physical constants of the system are described and analyzed in this report, and the results of one determination of the constants are used for illustration.

TABLE OF SYMBOLS

A_d Diaphragm area (effective) (cm^2)

A_m Manometer cross-sectional area (cm^2)

A_t Cross-sectional area of glass tube (cm^2)

B Magnet flux density (gauss)

D Linear displacement of water level in the manometer or measuring tube

E Precision of hydrophone calibration

e Voltage (volts)

F Force (on the coil-diaphragm by the magnetic field)(dynes)

f Frequency (sec^{-1})

g Gravitational constant (980 cm/sec^2)

h Linear displacement of mercury level in manometer (cm)

i Current (abamp or amp)

K Attenuator factor $\dfrac{10RR_2}{R+R_1+R_2}$

L Voice coil wire length (cm)

M_d Diaphragm mass (effective)(grams)

P Pressure (microbars or $dynes/cm^2$)

R, R_1, R_2 Attenuator circuit resistances

r Radius (cm)

S_c Chamber stiffness (effective at the diaphragm)(dynes/cm)

S_d Diaphragm stiffness (dynes/cm)

T Attenuator voltage attenuation (db)

T' Attenuator voltage attenuation (ratio)

V Volume displacement (cm^3)

Δ Change in

ϵ_x Error in hydrophone calibration due to error in x

η Hydrophone calibration [20 log (e_H/P)]

η' Hydrophone calibration (e_H/P)

ξ_d Linear displacement of diaphragm dome (cm)

ξ_m Linear displacement of diaphragm through manometer compliance (cm)

ρ Density of mercury (13.5 grams/cm^3)

ω Angular frequency ($\omega = 2\pi f$)(radians/sec)

$\approx$ Approximately equal to

$\stackrel{?}{=}$ Should be made equal to

log Logarithm to the base 10

$x = f(y,z)$ x is a function of y and z

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THE CALIBRATION OF A LOW-FREQUENCY CALIBRATING SYSTEM

1.0 INTRODUCTION

The low-frequency system used at the Underwater Sound Reference Laboratory was designed by the Bell Telephone Laboratories. Its general features are described in the *Summary Technical Report of Division 6, NDRC*, Vol. 10, pages 115-118. It is also described in detail in OSRD report 6.1 sr 783-1308, entitled *A Low Frequency Hydrophone Calibration System*. This latter report includes theory, development details, and operating instructions, and is prerequisite to a full understanding of the material presented in this report.

It is the purpose of this report to elaborate on and describe the analysis and techniques used in a recent calibration of this system at the USRL. By calibration of the system is meant the evaluation of the physical constants of the system which are used in obtaining the hydrophone calibration data. The details of this procedure are discussed only briefly in the OSRD report.

Fig. 1 shows all the system components important to the system calibration.

2.0 GENERAL ANALYSIS

2.1 How the System Functions

After the low-frequency system is calibrated, the hydrophone calibration is obtained in three simple steps:

(a) The output voltage e_A from the attenuator is made equal to the hydrophone output voltage e_H by adjustment of the attenuator (R_2).

(b) The hydrophone calibration is read directly from the attenuator dial.

(c) Corrections for chamber stiffness S_c, and hydrophone preamplifier-coupling gain or loss, are applied.

2.2 Fundamental Equations

The equations which relate the hydrophone calibration to the attenuator setting, and the mercury displacement in the manometer to the chamber stiffness are as follows:

$$\eta = 20 \log_{10} \left[\frac{10RR_2A_d(S_c+S_d-\omega^2M_d)}{BL(R+R_1+R_2)S_c} \right] - T \qquad (2.1)$$

$$S_c = \frac{2k\rho g A_d}{\xi_d - \xi_m} \qquad (2.2)$$

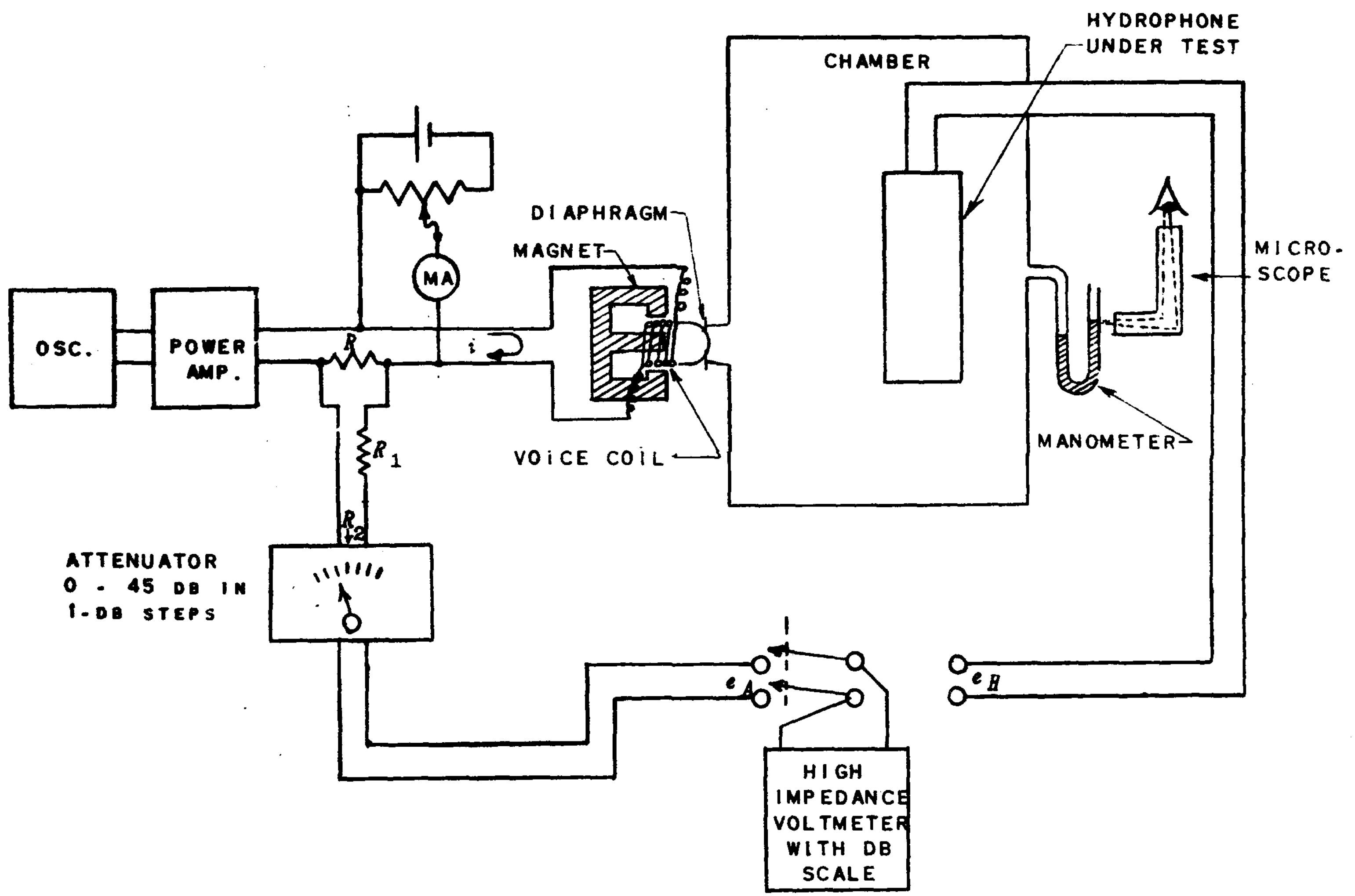


Fig. 1. Calibration System

$$\xi_d = \frac{0.1BLi - 2h\rho g A_d}{S_d} \tag{2.3}$$

$$\xi_m = h\left(\frac{A_m}{A_d}\right) \tag{2.4}$$

These equations are derived in the OSRD report. The individual terms in these equations may be grouped as follows:

(a) System constants, S_d, M_d, A_d, A_m, B, L, R, R_1, R_2, and T, which are functions of the electrodynamic projector, manometer, or attenuator circuit and which are assumed to remain unchanged unless the system components are altered. T is an "adjustable" constant.

(b) System variable, S_c, which is a function of the type of hydrophone under test, the air bubbles and other pressure-release material in the tank, and the temperature of the system.

(c) Universal constants, ρ and g.

(d) Variables, i, h, and ω.

(e) Values that cancel, ξ_d and ξ_m.

2.3 Accuracy

2.31 Independent Methods

The measurement of each physical constant was made by more than one method, and, insofar as it was possible, each method was made independent of the others. Frequently two methods had a common factor, but an error in such a factor was usually subject to cancellation. Generally, the arithmetic average of the data from different methods was taken as the final result, unless there was good reason to give more weight to particular data. The deviation will be defined as the difference between an accepted final average value, and the value determined by a single method.

2.32 Interdependence of the Constants

To a large extent the value of a constant, once measured, was used in the measurement and computation of a different constant. This interdependence is helpful in cross-checking the final results providing circular reasoning is not used; that is, one cannot measure A in terms of B, B in terms of C, and C in terms of A. How helpful the interdependence can be is best illustrated by the measurement of S_d. Two volume displacement methods were used—one dependent on BL and one independent. The fact that these two methods show good agreement implies that BL is probably correct.

2.33 Estimated Errors

After each constant was measured, an estimate was made of the probable error. These errors were then used in paragraph 13.3 to ascertain the probable precision of the over-all calibration.

3.0 MEASUREMENT OF THE FORCE FACTOR BL

3.1 Analysis

Although B and L are separate constants, only their product is used in the equations. The product BL is frequently called the "force factor" since it represents the force on a wire in a magnetic field as a function of the current in the wire ($F = BLi$) This force factor can be readily measured. BL is therefore taken as a single constant

3.2 Methods

3.21 General Considerations

A convenient way to measure BL is to place the voice coil in the magnetic gap, drive a measured direct current through the coil, and measure the force exerted on the coil with a weight scale or balance The force and current are then related by the following equations:

$$F = BLi \quad (F \text{ in dynes, } i \text{ in abamps})$$

$$980F = BLi/10 \quad (F \text{ in grams, } i \text{ in amps})$$

$$BL = 9800\ F/i$$

A common trip balance, accurate to ±0 1 gram is sufficient to measure the force This principle was used in the three slightly different methods described below. The apparatus is shown schematically in Fig. 2

A fourth, "floating", method is also described below.

3.22 The Fish Line Method

The diaphragm and coil assembly was suspended from the edge of a trip balance pan with fish line and tape. The magnet was placed such that the coil hung in its normal position in the magnetic field when the trip balance was in balance Particular care was taken to assure a free-moving system, although friction to the extent of about 2 gram weights persisted.

A measured direct current was passed through the coil in such a direction as to pull it down. The system was balanced while the current was flowing If the system were free of friction, the apparent weight of the diaphragm when the system was balanced would have provided the required data; however, the presence of the 2 gram weights of friction necessitated additional measurement. With the system in balance, the pull of the trip balance was *increased* in increments of a few tenths of a gram until the voice coil was pulled out of the magnetic gap. This release is sudden and not gradual. The procedure was repeated except that the pull of the balance was *decreased* until the voice coil was pulled down into the magnet. The balance adjustment was computed at these two extremes and the average taken as the correct apparent weight of the diaphragm assembly.

The pull of the balance can be changed either by readjustment of the weights in the balance arm or by adding to, or subtracting from, a group of analytical balance weights in the balance pan The second of these methods is preferred since changes can be made with least disturbance to the balanced system.

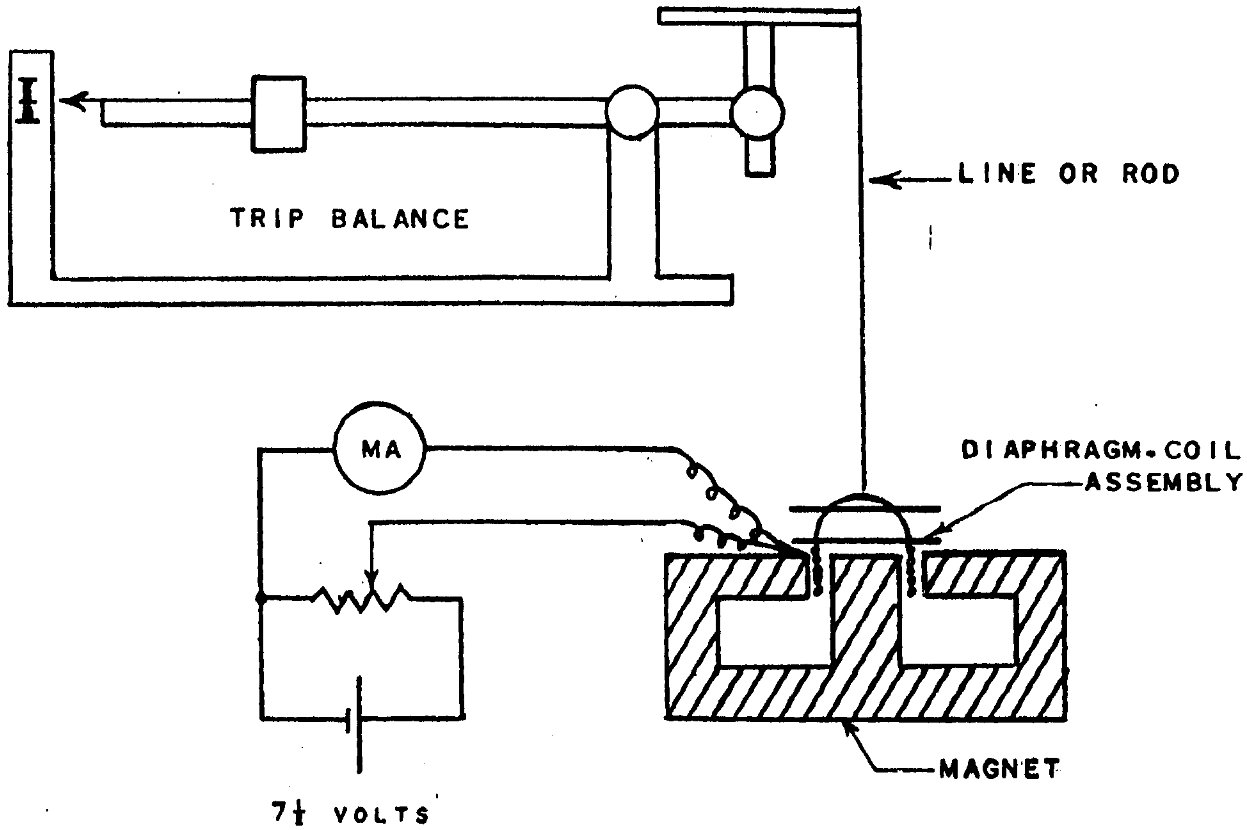


Fig. 2. *BL* Measurement Apparatus

The weight of the diaphragm assembly alone was also measured, although this was not necessary since the method of differences, which cancels out such constant factors, was used, as is discussed in paragraph 3.3.

Six different currents were used and the corresponding apparent weights computed. The data are shown in Fig. 3.

3.23 The Pull Rod Method

The same general scheme was used here as in the Fish Line method, except that the fish line was replaced with a 1/16-inch brass rod soldered to the diaphragm dome and to the edge of the balance pan. This provided a rigid, and generally more satisfactory, connection. It also allowed the coil either to pull or push the trip balance. In the Pull Rod method, the voice coil current pulled down just as in the Fish Line method. The data are shown in Fig. 4.

3.24 The Push Rod Method

In this method the current was passed through the voice coil in such a direction as to cause the coil to push up. This permitted a balancing out of the weight of the diaphragm assembly by the trip balance arm adjustment before current was driven through the coil. Then, with current through the coil, the system was brought into balance by adding weights to the balance pan in an amount sufficient to equalize the apparent negative weight of the coil. After the balanced system condition was obtained, procedures for cancelling out the frictional drag, as in the first two methods, were followed. Here the friction margin was ±1 gram instead of the ±2 grams found in the Fish Line and Pull Rod methods. The data are shown in Fig. 4.

3.25 The Floating Method

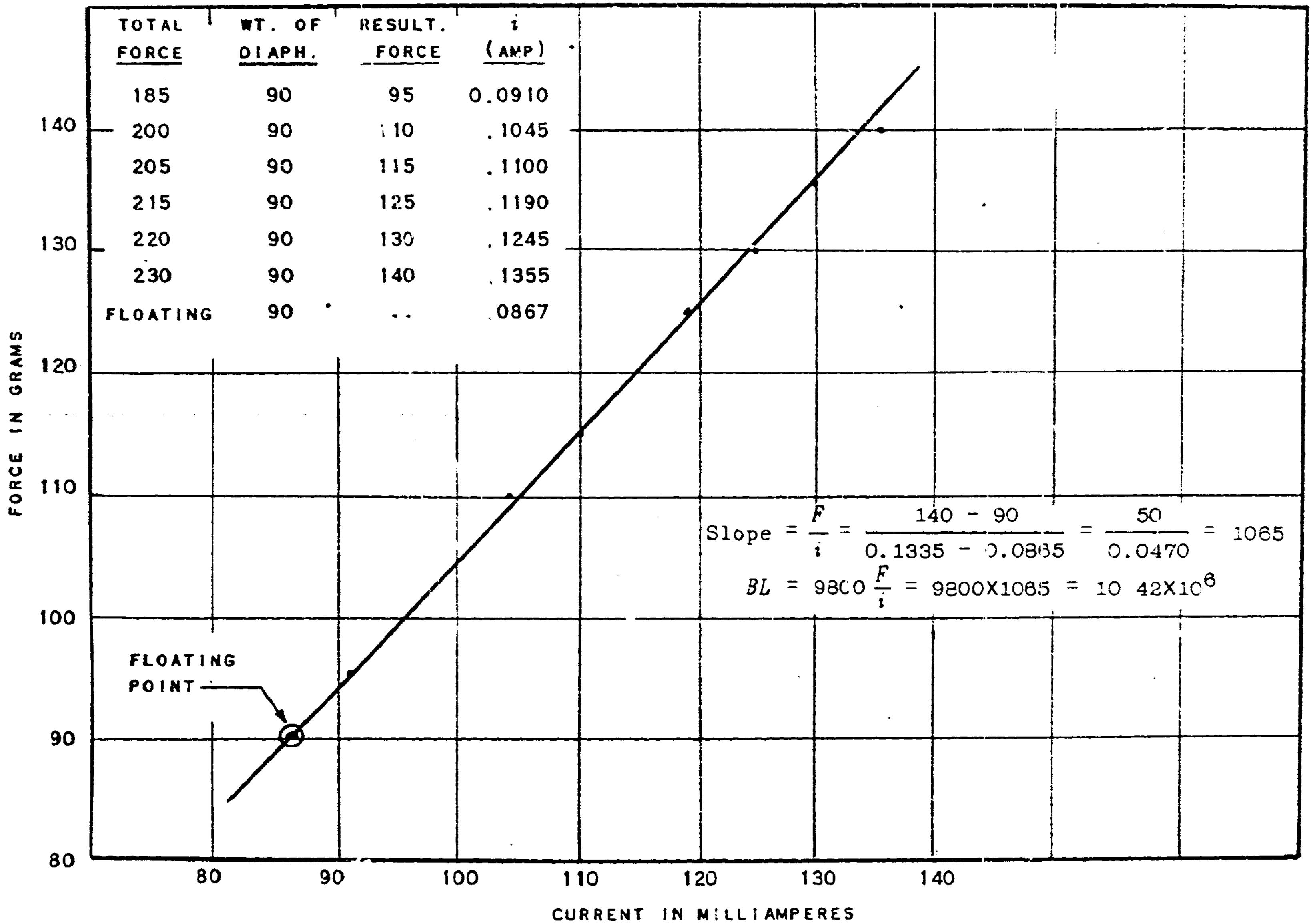
As a single point check, the current was found which would cause the diaphragm assembly to float in the magnetic gap. Under this condition, the force would be equal and opposite to the diaphragm assembly weight. This was done two ways—by floating the diaphragm above the magnet, and by hanging the diaphragm below the inverted magnet. The current in each case was the same. This single check point in each case is shown in Fig. 3.

3.26 Other Methods

Other methods, involving the use of a fluxmeter and the measurement of small displacements, can be used. These methods have advantages over the force factor method, but were not used here because of the temporary lack of proper equipment.

3.3 Data Summary

In order to eliminate errors in absolute measurement, the method of differences was used. That is, the slope of the curve $F = f(i)$ was found. This eliminated constant errors in both F and i. The data taken by the four methods had a total spread of 0.37×10^6. The average value of BL, giving all four methods equal weight was 10.2 gauss-cm. The summary data are tabulated below.



TOTAL FORCE	WT. OF DIAPH.	RESULT. FORCE	i (AMP)
185	90	95	0.0910
200	90	110	.1045
205	90	115	.1100
215	90	125	.1190
220	90	130	.1245
230	90	140	.1355
FLOATING	90	..	.0867

Fig 3. Data for fishline and floating methods

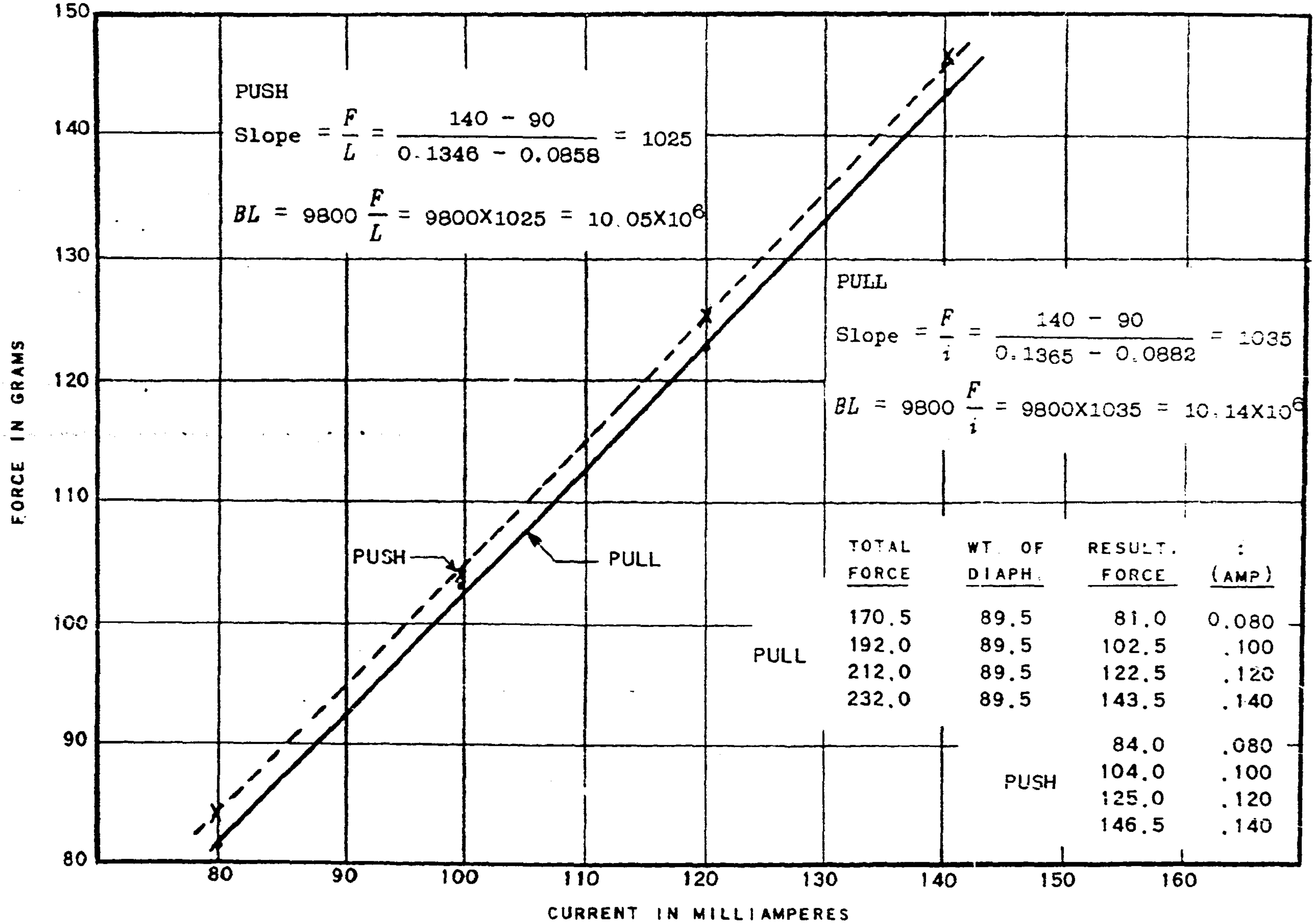


Fig. 4. Data for pull rod and push rod methods

METHOD	BL	DEVIATION
Fish Line	10.42×10^6	$+0.22 \times 10^6$
Pull Rod	10.14×10^6	$-.06 \times 10^6$
Push Rod	10.05×10^6	$-.15 \times 10^6$
Floating	10.17×10^6	$-.03 \times 10^6$

Average 10.20×10^6 ($\pm 0.20 \times 10^6$) gauss-cm

4.0 MEASUREMENT OF THE DIAPHRAGM STIFFNESS S_d

4.1 Analysis

The diaphragm stiffness is defined as

$$S_d = F/\xi_d$$

where F = force on the diaphragm in dynes;

ξ_d = linear displacement, in centimeters, of the equivalent piston diaphragm (see paragraph 5.1 for discussion of "equivalent piston diaphragm").

Measurement of S_d requires a means of applying a known force to the diaphragm, and measuring the linear displacement, ξ_d. A known force can be obtained by driving a known current through the voice coil, and using the known force factor BL to calculate the force on the coil-diaphragm assembly. As an alternative, if the effective area of the diaphragm A_d is known (see paragraph 5.3), air pressure can be used to move the diaphragm.

To measure the linear displacement directly, a dial indicator, accurate to 0.0001 inch can be used. Because of the small amplitude of the diaphragm motion, however, it is more accurate to measure the linear displacement indirectly by measurement of the volume displacement. If the volume displacement of the whole diaphragm (effective area ≈ 14 cm^2) is channeled into and measured in a small tube (cross-sectional area = 0.1 cm^2), the linear displacement is magnified by a factor of 140, and thus made easier to measure. Here again, A_d must be known to correlate the volume displacement and linear displacement.

With two methods of moving the diaphragm (coil current or air pressure), and two methods of measuring displacement (linear or volume) four different combinations can be used in measuring S_d. All will be dependent on other known constants (BL, A_d, and A_m), but to different degrees or in different ways, so that each method will be partially or totally independent of the others.

The stiffness of the diaphragm can also be found by the resonant method if the effective diaphragm is known.

Further analysis details are given under the individual methods used.

4.2 Methods

4.21 S_d as a Function of Volume Displacement and Voice Coil Current, $S_d = f(V, i)$.

4.211 Description

The schematic arrangement for measuring S_d as a function of the voice coil current and volume displacement is shown in Fig. 5. The function is:

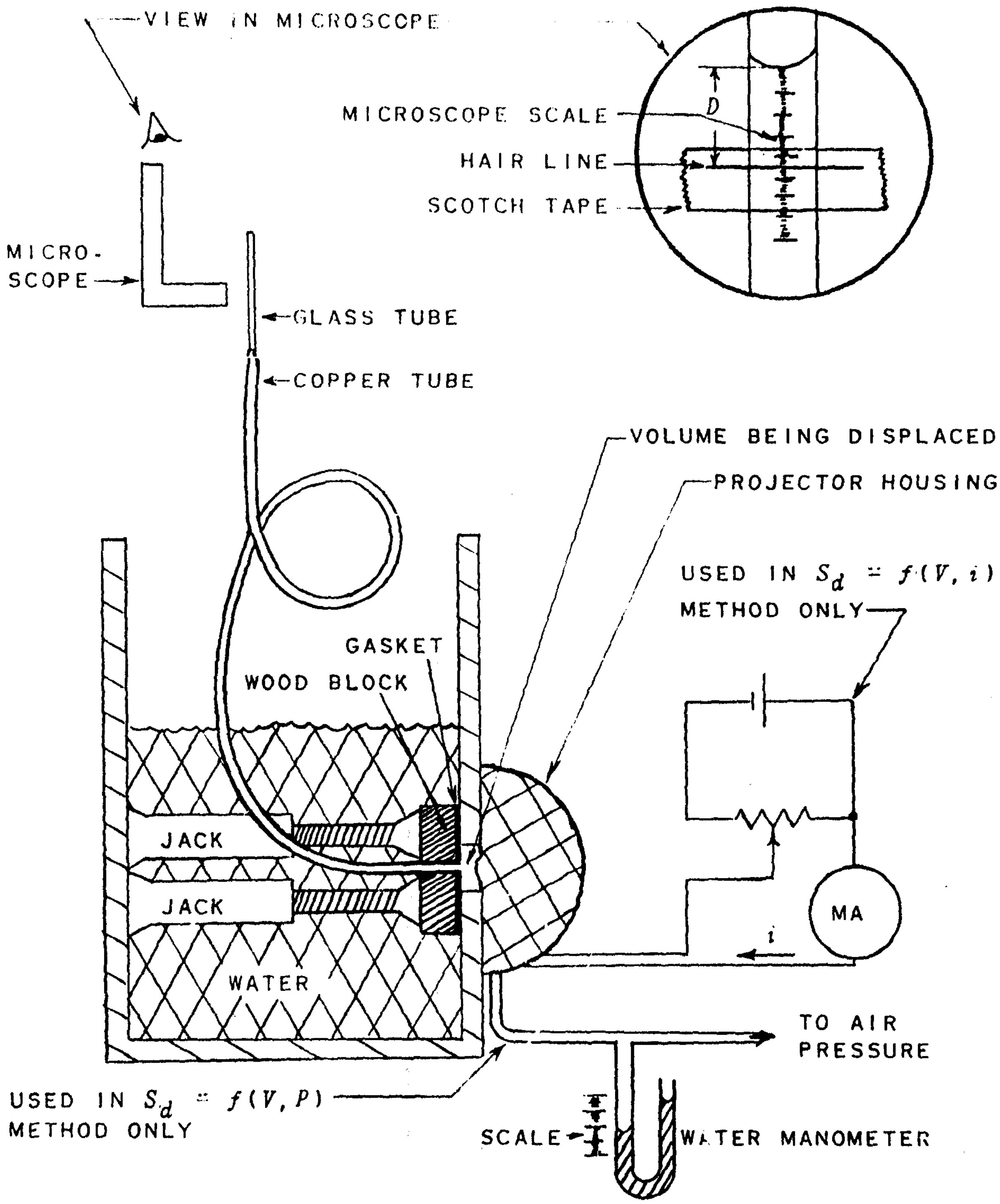


Fig. 5. Volume displacement apparatus

$$S_d = \frac{F}{\xi_d} = \frac{BLi}{V/A_d} = BLA_d \frac{i}{V}$$

The volume displacement is measured in the glass tube, of cross-sectional area A_t, by measuring the linear displacement D of the water level. That is

$$V = A_t D$$

$$\therefore S_d = BLA_d \left(\frac{i}{V}\right) = \frac{BLA_d}{A_t} \left(\frac{i}{D}\right)$$

From other measurements.

$$BL = 10.2\text{X}10^6 \text{ gauss-cm (para. 3.3)}$$
$$A_d = 13.8 \text{ cm}^2 \text{ (para 5.3)}$$
$$A_t = A_m = 0.098 \text{ cm}^2 \text{ (para 7.0)}$$
$$\therefore S_d = 1465\text{X}10^6 \ (i/D) \text{ dynes/cm}$$

Constant errors in both i and D can be cancelled out by taking the reciprocal of the slope of the curve $D = f(i)$ as is shown in the data in Fig. 6.

4.212 Procedure Notes

Three points in this technique require elaboration:

(a) It was necessary to keep the volume undergoing displacement as small as possible. The whole tank was first tried, but the changing volume, due to temperature changes, caused the level in the measuring tube to drift too rapidly to obtain data. The volume was drastically reduced by putting the block fitting over the small recess in the tank wall in front of the diaphragm (see Fig. 5).

(b) It is important to keep the volume displacement through other parts of the system (e.g., rubber gasket) constant. Therefore, the static pressure must be constant. This was done by making the copper tubing compliant and adjustable in the vertical direction. Before any data were taken, the water level was adjusted to a fixed position by aligning it with the zero point on the microscope scale. After a level change due to a volume displacement induced by the moving diaphragm, the level was again adjusted to zero before the linear displacement data were taken. These data were read as the displacement of a hair line on the glass tube with respect to the microscope scale. The hair line on the glass tube was literally a hair held in place with Scotch tape. Since the change in displacement D with change in current i were the important data, the initial relative position of the hair line with respect to the water level was unimportant.

(c) The whole block and tubing assembly had to be assembled under water to avoid trapped air in the volume being displaced.

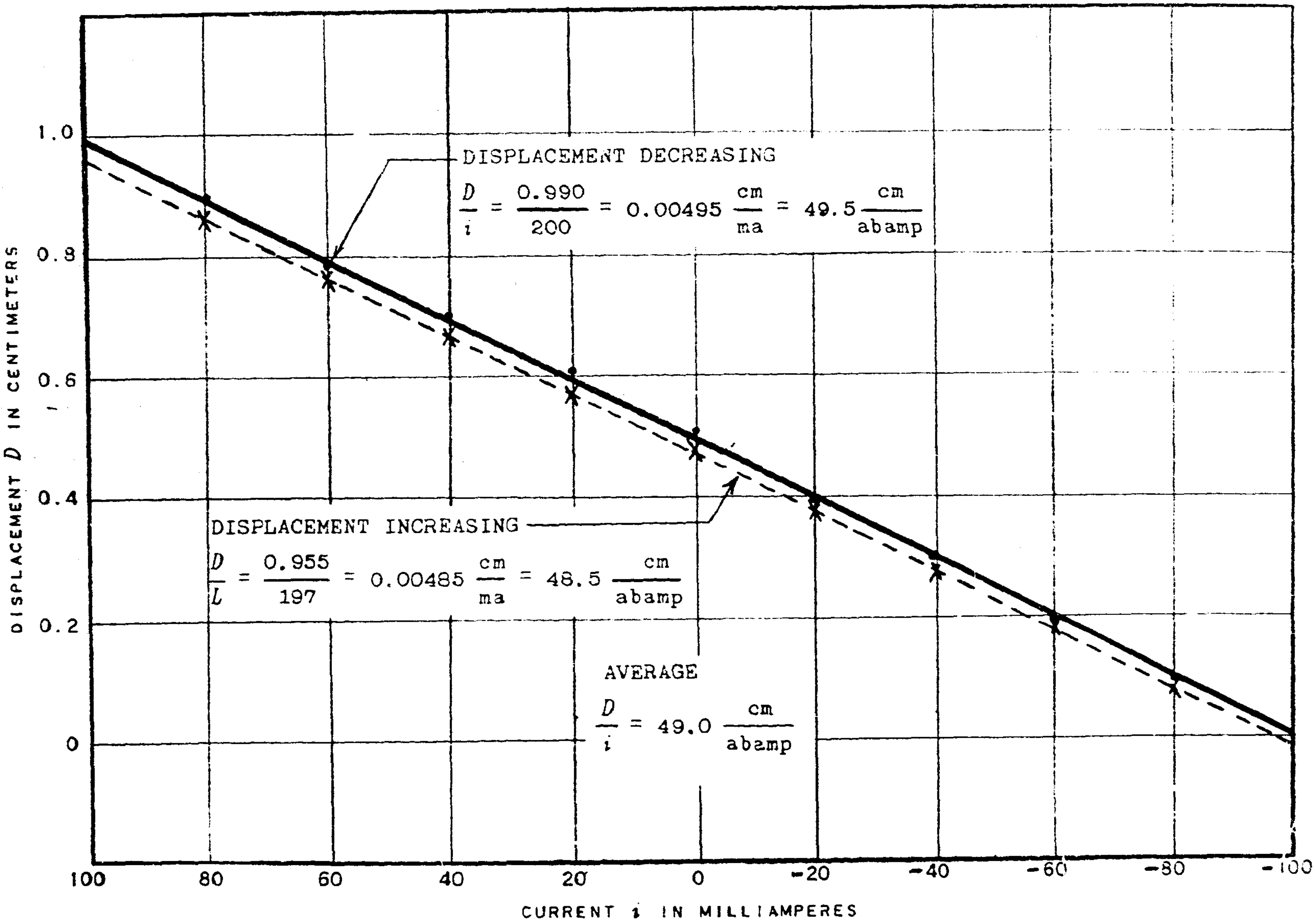


Fig. 6 Linear displacement D of water level in the glass measuring tube, as a function of voice coil current i.

4.213 Data

The data from this method are shown in Fig. 6

4.22 S_d as a Function of Volume Displacement and Pressure, $S_d = f(V,P)$

4.221 Description

The schematic arrangement for measuring S_d as a function of the volume displacement and the pressure on the diaphragm is shown in Fig. 5. The function is

$$S_d = \frac{F}{\xi_d} = \frac{PA_d}{V/A_d} = A_d^2 \left(\frac{P}{V}\right)$$

Just as in the $S_d = f(V,i)$ method, the volume displacement $V = A_t D$,

$$\therefore S_d = A_d^2 \left(\frac{P}{V}\right) = \left(\frac{A_d^2}{A_t}\right)\left(\frac{P}{D}\right)$$

From other measurements

$$A_d = 13.8 \text{ cm}^2$$

$$A_t = A_m = 0.096 \text{ cm}^2$$

$$\therefore S_d = 1985(P/D) \text{ dynes/cm}$$

The procedures for measuring the volume displacement as a function of the linear displacement D in the measuring tube were the same here as in the first method. A water manometer was used to measure the pressure changes because such changes were too small to be measured with a mercury manometer or conventional gage.

4.222 Data

The data obtained with this method are shown in Fig. 7. Note that the change in level in one arm of the manometer represents one-half the total change in pressure.

4.23 S_d as a Function of Force and Linear Displacement, $S_d = f(F,\xi)$

4.231 Description

The schematic arrangement for measuring S_d as a function of the linear displacement of the rigid or dome part of the diaphragm, and of the force F is shown in Fig. 8. As is shown in Fig. 9, this displacement, and the displacement of the equivalent piston diaphragm are the same. The function is

$$S_d = \frac{F}{\xi_d} = \frac{BLi}{\xi_d} \text{ (}i\text{ in abamps)} = \frac{BLi}{10\xi_d} \text{ (}i\text{ in amps)}$$

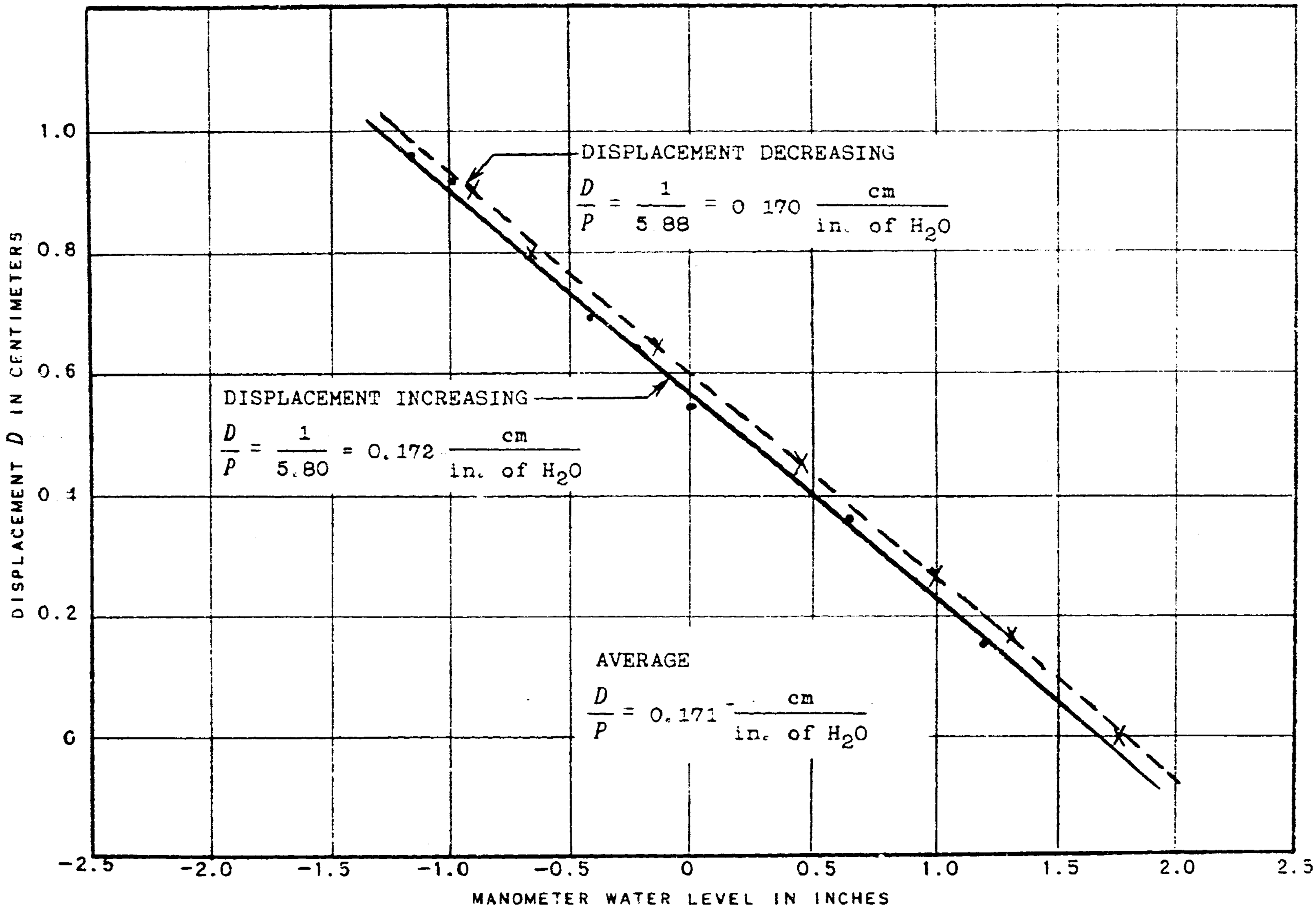
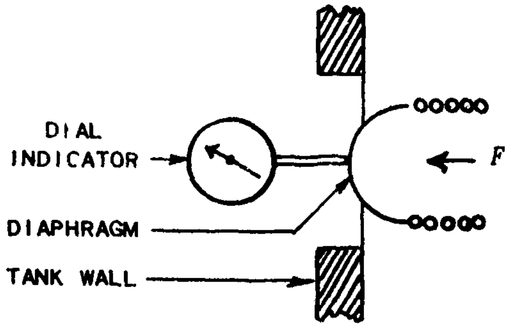


Fig. 7. Linear displacement D as a function of air pressure on back of diaphragm

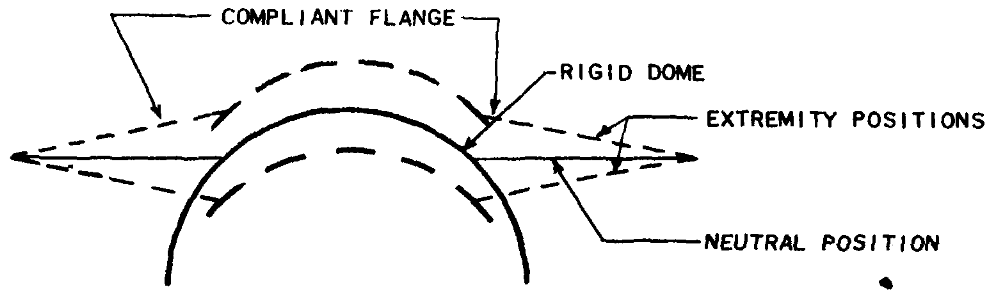


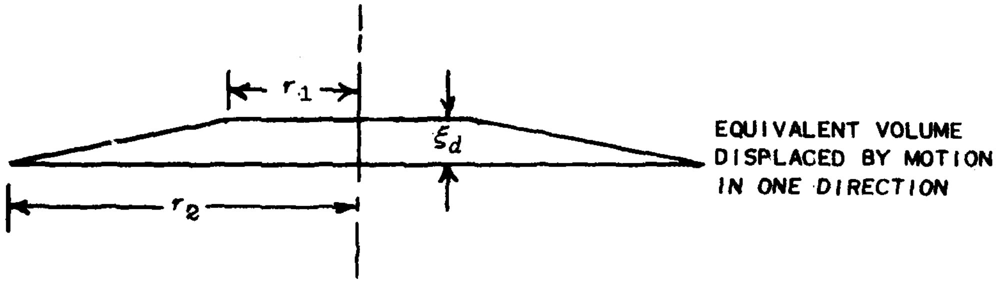
i COIL CURRENT IN MA	DIAL READING IN .001 IN. i ON	i OFF	DIAL READING CHANGE	$\frac{D}{L}$ $\frac{(.001 \text{ in.})}{(\text{amps})}$
+200	-3.2	-0.6	2.6	13.0
+200	-3.2	-0.6	2.6	13.0
-200	-3.6	-6.2	2.6	13.0
-200	-3.6	-6.2	2.6	13.0
-200	-3.6	-6.2	2.6	13.0
-100	-3.5	-4.8	1.3	13.0
-100	-3.5	-4.8	1.3	13.0
-100	-3.5	-4.8	1.3	13.0
+100	-3.2	-1.9	1.3	13.0
+100	-3.2	-1.8	1.2	12.0
+100	-3.2	-1.8	1.2	12.0
+100	-3.2	-1.8	1.2	12.0
-100	+1.0	-0.3	1.3	13.0
-100	+0.9	-0.3	1.2	12.0
-100	+1.0	-0.3	1.3	13.0
-100	+1.0	-0.3	1.3	13.0
+200	-0.2	+2.4	2.6	13.0
+200	-0.1	+2.5	2.6	13.0
+200	0	+2.6	2.6	13.0
+200	0	+2.6	2.6	13.0
-200	+3.5	+6.0	2.5	12.5
-200	+3.5	+6.1	2.6	13.0
-200	+3.5	+6.1	2.6	13.0
-200	+3.5	+6.1	2.6	13.0

DIAL INDICATOR ADJUSTMENT CHANGED

$$\frac{307.5}{24} = 12.81 \approx 13$$

Fig. 8. Diaphragm displacement as a function of voice coil current.





$$\text{Vol.} = \frac{\pi}{3}\,(r_1^2 + r_1 r_2 + r_2^2)\,\xi_d$$

$$A_d = \frac{\text{Vol.}}{\xi_d} = \frac{\pi}{3}\,(r_1^2 + r_1 r_2 + r_2^2)$$

Note: ξ_d is the linear displacement of both the diaphragm dome and the equivalent piston diaphragm.

$$\text{RADIUS OF EQUIVALENT PISTON} = \sqrt{\frac{A_d}{\pi}} = \sqrt{\frac{1}{3}\,(r_1^2 + r_1 r_2 + r_2^2)}$$

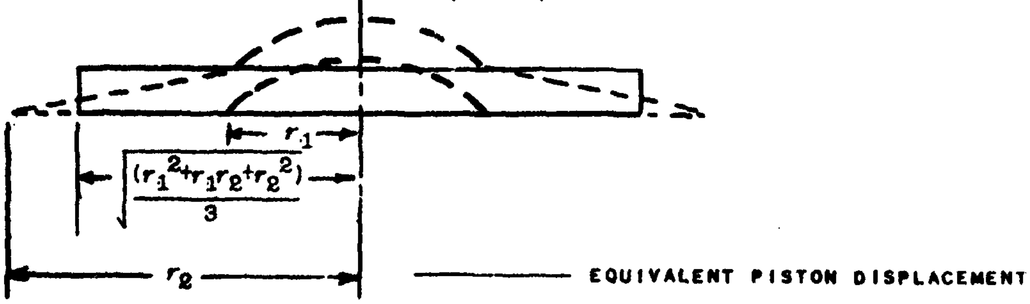


Fig. 9. Diaphragm displacement geometry

From other measurements

$$BL = 10.2 \times 10^6 \text{ gauss-cm}$$

$$\therefore S_d = 1.02 \times 10^6 (i/\xi_d) \text{ dynes/cm}$$

4.232 Data

The data from this method are shown in Fig. 8. Unlike the other methods, small increments of current and force could not be used here because either the diaphragm or the dial indicator tended to jerk. Therefore, only four currents were used. To compensate for the inability to draw a curve and measure the slope as in other methods, the data were repeated several times. Note that the current was repeatedly switched on and off, and the corresponding displacement jumps read on the dial indicator. This method proved the most consistent of several tried.

4.24 S_d as a Function of Pressure and Linear Displacement, $S_d = f(P, \xi)$

The last of the four possible methods determined S_d as a function of pressure and linear displacement. That is

$$S_d = \frac{F}{\xi_d} = \frac{PA_d}{\xi_d} = A_d \left[\frac{P}{\xi_d}\right]$$

This method failed to produce usable data. The combination of the jerkiness mentioned in paragraph 4.232 and the inability to obtain precise control of the air pressure led to the abandonment of this method.

4.25 Resonant Method

In paragraph 6.1 it is shown that the resonant frequency of the diaphragm is given by

$$f = \frac{1}{2\pi} \sqrt{\frac{S_d}{M_d}}$$

This equation is used to find M_d, given S_d. However, if M_d is measured by a means independent of S_d, such a value could be used here to find S_d. Such a value is given in paragraph 6.222. The resonant frequency is found by noting the frequency at which the impedance peaks or the current through the coil dips.

When $M_d = 15.5$ grams and $f = 222$ cps

$$S_d = M_d(2\pi f)^2 = 15.5(2\pi)^2(222)^2 = 30.2 \times 10^6 \text{ dynes/cm}$$

4.3 Data Summary

The four methods of measuring S_d provide the final data:

	S_d	DEVIATION
$S_d = f(V, i)$	29.9×10^6	-0.1×10^6
$S_d = f(V, P)$	28.9×10^6	-1.1×10^6
$S_d = f(F, \xi)$	30.9×10^6	$+0.9\times10^6$
$S_d = f(M_d, f)$	30.2×10^6	$+0.2\times10^6$
	Average 30.0×10^6 ($\pm1.0\times10^6$) dynes/cm	

5.0 MEASUREMENT OF THE EFFECTIVE AREA OF THE DIAPHRAGM A_d

5.1 Analysis

The symbol A_d is used to represent the area of an equivalent piston diaphragm; that is, a diaphragm which moves like a piston, and whose area relates the force and pressure on the diaphragm which produce the same volume displacement. That is

$$V = \frac{FA_d}{S_d} = \frac{PA_d^2}{S_d} = \xi_d A_d$$

The linear displacement ξ_d is also defined by the above relationship. Two totally independent methods were used to measure A_d. In one, the geometry of the diaphragm was used; in the other, the relationship between the force and pressure exerted on the diaphragm was used.

5.2 Methods

5.21 Diaphragm Geometry

5.211 Description

Inasmuch as the amplitude of vibration is small compared to the radial dimensions of the diaphragm, the assumption is made that the volume displacement, in one direction, is geometrically equivalent to the volume of the frustum of a right circular cone. This is schematically shown in Fig. 9. With this assumption, A_d can be calculated from the linear measurements of the diaphragm. This calculation is also shown in Fig. 9.

The measurement of the radii r_1 and r_2 was made with vernier calipers after careful visual examination of the diaphragm to ascertain the inner and outer edges of the compliant flange part of the diaphragm.

5.212 Data

The radii were measured

r_1 = 16.5 mm ±0.5 mm

r_2 = 25.4 mm ±0.5 mm

$$\therefore A_d = \frac{3.14}{3}\,(16.5^2 + 16.5 \times 25.4 + 25.4^2)$$

$$A_d = 14.0 \text{ cm}^2$$

5.22 A_d as a Function of the Force-to-Pressure Ratio, $A_d = f(F/P)$

5.221 Description

From the definition of A_d in paragraph 5.1

$$V = \frac{FA_d}{S_d} = \frac{PA_d^{\,2}}{S_d}, \quad \text{or } A_d = \frac{F}{P}$$

The terms F and P here are the force and pressure which produce the same volume displacement. Such data are available from the diaphragm stiffness measurements.

5.222 Data

The stiffness measurement data in Figs. 6 and 8 may be used with the following identities:

$$A_d = \frac{F}{P} = \frac{BLi}{P} = BL\left(\frac{i}{D}\right)\left(\frac{D}{P}\right)$$

From Fig. 6, $\quad \dfrac{i}{D} = \dfrac{1}{49.0}$ abamp/cm

From Fig. 7, $\quad \dfrac{D}{P} = \dfrac{6.87}{10^5}$ cm/microbar

From other measurements, $BL = 10.2 \times 10^6$ gauss-cm

$$\therefore A_d = 10.2 \times 10^6 \left(\frac{1}{49.0}\right)\left(\frac{6.87}{10^5}\right) = 13.72 \text{ cm}^2$$

Using data which repeated those in Figs. 6 and 7, but which are not included in this report because D was incorrectly measured (it cancels out here), another value of A_d is obtained

$$A_d = 13.9 \text{ cm}^2$$

5.3 Data Summary

The measurements in the $A_d = f(F/P)$ method are estimated to be much more accurate than the geometry method. Therefore, greater weight has been given to the two values of A_d obtained from the second method

			DEVIATION
A_d	$f(r_1, r_2)$	14.0	+0.2
A_d	$f(F/P)$	13.7	-0.1
		13.9	+0.1

Weighted average 13.84 cm^2

Accepted average 13.8 (± 0.2) cm^2

6.0 MEASUREMENT OF THE EFFECTIVE DIAPHRAGM MASS M_d

6.1 Analysis

The mechanical circuit of the diaphragm is a simple series circuit as shown in Fig. 10

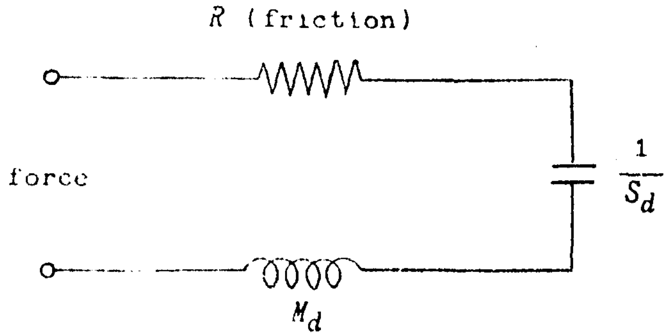


Fig. 10. Mechanical circuit of diaphragm

Its resonant frequency is given by the familiar equation

$$f = \frac{1}{2\pi\sqrt{M_d(1/S_d)}} = \frac{1}{2\pi}\sqrt{\frac{S_d}{M_d}}$$

This equation defines M_d in terms of S_d and f. The equation also provides two methods of finding M_d. The first is by simple calculation, given S_d and f. The second is by changing M_d by a known amount and noting the change in f.

That is, $$f_0 = \frac{1}{2\pi}\sqrt{\frac{S_d}{M_d}} \qquad (6.1)$$

Now add a mass ΔM to M_d, and let the new resonant frequency $f_0 + \Delta f = f_1$

Then $$f_1 = \frac{1}{2\pi}\sqrt{\frac{S_d}{M_d+\Delta M}} \qquad (6.2)$$

Dividing (6.1) by (6.2) and squaring produces

$$\frac{f_0^2}{f_1^2} = \frac{M_d + \Delta M}{M_d} = 1 + \frac{\Delta M}{M_d}$$

$$\therefore M_d = \frac{\Delta M}{(f_0/f_1)^2 - 1} \tag{6.3}$$

In an electrodynamic driver, as we have here, the mechanical impedance appears as an electrical conductance at the electrical terminals. Therefore, at mechanical resonance the mechanical impedance dips, but the electrical impedance peaks. With a constant voltage or constant power source, the current will then dip sharply. In this low-frequency system, the voltage at the attenuator output is proportional to the driving current, and thus can be used to find the resonant frequency.

6.2 Methods

6.21 M_d as a Function of S_d and the Resonant Frequency, $M_d = f(S_d, f)$

Using the attenuator output voltage as an indicator, the resonant frequency of the diaphragm was found to be 222 cps. From paragraph 4.3 S_d was taken as 30×10^6. Using these values in the equation

$$f = \frac{1}{2\pi}\sqrt{\frac{S_d}{M_d}}$$

or

$$M_d = \frac{S_d}{(2\pi f)^2}$$

we have

$$M_d = \frac{30 \times 10^6}{(2\pi 222)^2} = 15.4 \text{ grams}$$

6.22 M_d as a Function of Changes in Diaphragm Mass and the Resonant Frequency, $M_d = f(\Delta M, \Delta f)$

6.221 Description

To use equation (6.3) derived in paragraph 6.1, a known mass must be added to the diaphragm in a manner such that ΔM and M_d continue to act like a single lumped constant. The method used here was to mount small lead slugs on the dome of the diaphragm using a sticky material (Korite) for an adhesive.

The data obtained appeared accurate and consistent until ΔM became greater than M_d.

The attenuator output voltage was again used as an indicator to locate the resonant frequencies.

The oscillator frequency scale required careful and frequent calibration because of persistent drifting This drifting was believed to be caused by temperature changes in the oscillator The oscillator calibration was made by putting the oscillator frequency on the Y plates of an oscilloscope, and the 60-cps power frequency on the X plates. A sufficient number of Lissajous figures can be obtained this way to plot a calibration curve.

6.222 Data

The data obtained using the equation

$$M_d = \frac{M}{(f_0/f_1)^2 - 1}$$

are tabulated below:

FREQUENCY		ΔM		
DIAL	CORRECTED	(GRAMS)	M_d	DEVIATION
243	222	0	-----	-----
216	197	4.2	15.55	+0.05
185	169	11.2	15.45	- .05
179	164	13.0	15.60	+ .10
165	152	20.3	17.88	(not used)
		Average	15.5	grams

6.3 Data Summary

The two methods used show good agreement. The first was largely dependent on S_c and the resonant frequency. The second was dependent only on M_d. A constant error in frequency would cancel out. The methods are therefore more independent of each other than it would first appear.

$M_d = f(S_c, f) = 15.4$ grams

$M_d = f(\Delta M, \Delta f) = \underline{15.5}$ grams

Accepted value 15.4 (±0.2) grams

7.0 MEASUREMENT OF THE MANOMETER TUBE CROSS-SECTIONAL AREA A_m.

A_m can be determined by simple measurement of the tube diameter on the microscope scale. The inside diameter of the two ends of the manometer tube were examined in this manner. The diameter in different directions was noted. All diameters examined measured 0.350 cm ±0.001 cm. In addition, the outside diameters were examined at various points on the tube, and showed good consistency, and no distortion. No other type of measurement was made.

$$A_m = \pi r^2 = \pi\left(\frac{0.350}{2}\right)^2 = 0.0961 \text{ cm}^2$$

$$A_m = 0.096\ (\pm 0.002)\ \text{cm}^2$$

8.0 MEASUREMENT OF ATTENUATOR CIRCUIT CONSTANTS R, R_1, R_2, and T.

8.1 Analysis

The attenuator circuit constants relate the voltage e_A to the projector current i as shown in Fig. 11. This same relationship is used in equation (2.1). There is little reason for these constants to change from their original values, but they should nevertheless be checked. The easiest method of checking is to reduce the four constants to two, that is

(a) $$\frac{RR_2}{R+R_1+R_2}$$

(b) T (voltage attenuation in db)

or T' (voltage attenuation in numerical ratio)

where $20 \log T' = T$

In this way the constants are checked exactly as they are used in equation (2.1).

The original values are

$R = 23.5$ ohms

$R_1 = 575$ ohms

$R_2 = 600$ ohms

$T = 0 - 45$ db in 1-db steps

From the above

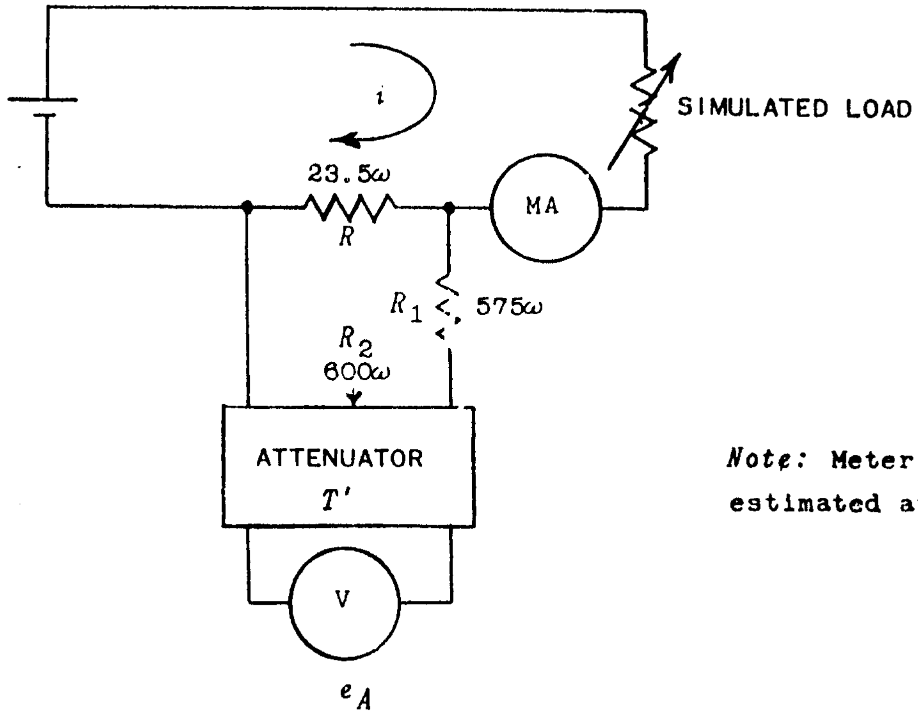
$$(a) = \frac{RR_2}{R+R_1+R_2} = 11.7$$

8.2 Methods

In order to provide more than one method of checking the constants, both a d-c and an a-c method were used.

8.21 D-c Method

The circuit shown in Fig. 11 was set up. A known direct current was supplied, and the voltage e_A measured. The measured e_A was then compared to the calculated e_A. The data are tabulated in Fig. 11.



Note: Meter accuracy estimated at ±3%.

$$\text{CALCULATED } e_A = \frac{iRR_2}{R+R_1+R_2}\,T' = i\,\frac{(23.5)(600)}{23.5+575+600}\,T' = 11.7iT'$$

	T'	i	MEASURED e_A	CALCULATED e_A	DIFFERENCE
(0 db)	1	90 ma	1.03 v	1.06 v	3%
	1	100	1.117	1.18	1%
	1	110	1.29	1.29	0
	1	120	1.42	1.42	0
	1	140	1.66	1.65	0.5%
	1	180	2.16	2.12	2%

Fig. 11. D-c attenuator circuit check

8.22 A-c Method

The circuit shown in Fig. 12 was set up. An alternating current was supplied. The current was calculated from e_i. The measured e_A was compared with the calculated e_A. The data are tabulated in Fig. 12.

8.23 Attenuator T

The data in Fig. 12 provide a check on the attenuator at zero and 6 db (86 on the attenuator dial). Beyond this, it is only necessary to note whether 1-db changes in the attenuator produce corresponding 1-db changes in the voltmeter shown in Fig. 12. This was done, and the attenuator range of 0 - 45 db in 1-db steps was verified.

8.3 Data Summary

All the data comparing the measured and calculated e_A showed good agreement. Therefore the value of (a) is accepted as

$$\frac{RR_2}{R+R_1+R_2} = 11.7\ (\pm 3\%)\ \text{ohms}$$

The attenuator constant T is accepted as ranging from zero to 45 db in 1-db steps. On the attenuator dial, this range is read as -80 to -125 db.

9.0 SUMMARY AND TABULATION OF SYSTEM CONSTANTS

9.1 Discussion

The values measured here are in general agreement with the original values given in the OSRD report, with the exception of BL. This factor is considerably smaller, probably because the projector housing had been flooded and the magnet rusted, requiring disassembly of the magnet for cleaning. This disassembly undoubtedly reduced the magnetic induction B by a considerable amount--from approximately 13,000 to 7,000 gauss, according to the available data.

9.2 Tabulation

$$BL = 10.2\times10^6\ (\pm 0.2\times10^6)\ \text{gauss-cm}$$

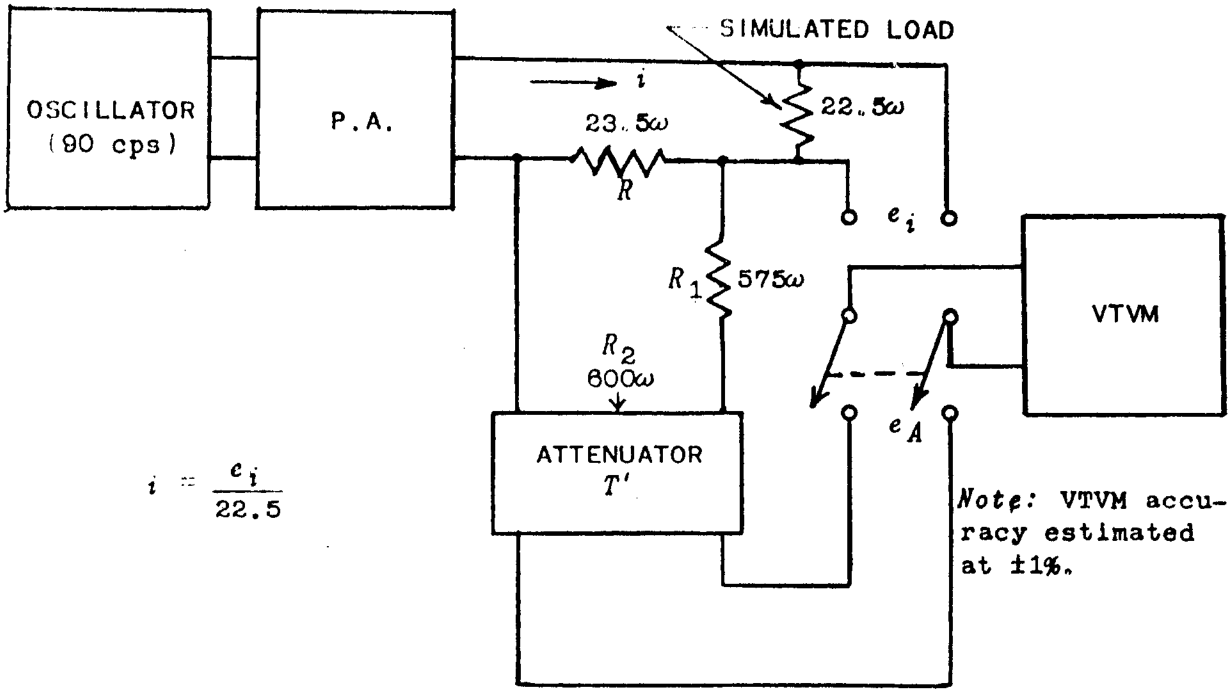
$$S_d = 30.0\times10^6\ (\pm 1.0\times10^6)\ \text{dynes/cm}$$

$$M_d = 15.4\ (\pm 0.2)\ \text{grams}$$

$$A_m = 0.096\ (\pm 0.002)\ \text{cm}^2$$

$$A_d = 13.8\ (\pm 0.2)\ \text{cm}^2$$

$$\frac{RR_2}{R+R_1+R_2} = 11.7\ (\pm 3\%)\ \text{ohms}$$



Note: VTVM accuracy estimated at ±1%.

$$i = \frac{e_i}{22.5}$$

$$\text{CALCULATED } e_A = \frac{iRR_2}{R+R_1+R_2}T' = i\,\frac{(23.5)(600)}{23.5+575+600}T' = 11.7iT'$$

	T'	e_i	i	MEASURED e_A	CALCULATED e_A	DIFFERENCE
(0 db)	1.0	2.25 v	100 ma	1.18 v	1.17 v	1%
(-6 db)	0.5	2.25	100	0.59	0.585	1%
	1.0	1.125	50	0.61	0.59	3%
	0.5	1.125	50	0.30	0.295	1.5%
	1.0	3.00	133.4	1.58	1.57	0.5%
	0.5	3.00	133.4	0.79	0.785	0.5%

Fig. 12. A-c attenuator circuit check

10.0 MEASUREMENT OF THE CHAMBER STIFFNESS S_c

S_c is defined as the chamber stiffness effective at the diaphragm. In effect, this means that S_c is smaller than the true chamber stiffness by a factor equal to the ratio of A_d to the chamber area (i.e., wall space). This concept is necessary to avoid measurement of the chamber area. From equations (2.2), (2.3), and (2.4)

$$S_c = \frac{2h\rho g A_d}{\left[(0.1BLi - 2h\rho g A_d)/S_d\right] - \left[h A_m/A_d\right]}$$

If we take $\rho = 13.5$ g/cm^3, $g = 980$ cm/sec^2, and insert the measured values from paragraph 9.2, S_c becomes

$$S_c = \frac{(1.51 \times 10^6)h}{0.141i - 0.0790h}$$

Thus S_c is measured as a function of the variables i and h.

At this point, it is advisable to review the operations involved in determining S_c. The system is closed and ready for a hydrophone calibration. A measured direct current is driven through the voice coil. The diaphragm moves out (or in) causing a volume displacement of the water. If the chamber is infinitely stiff, all the volume displacement will be passed through the mercury manometer, causing a linear displacement h_0 of the mercury level. However, the chamber is not infinitely stiff, and some volume displacement is shunted through such acoustic by-passes as the expanding chamber tank, a compressed air bubble, or a soft hydrophone. Thus, the linear displacement h of the mercury level will be something less than h_0 depending on the magnitude of S_c. The data is simplified if i is taken at a fixed value. Let $i = 200$ ma; then

$$S_c = \frac{(1.51 \times 10^6)h}{0.028 - 0.079h} \qquad (10.1)$$

11.0 FINDING η AS A FUNCTION OF S_c AND ω

The final step in the calibration of the system is to insert the values of the system constants in equation (2.1), and find η as a function of the two remaining variables h and ω, and the adjustable constant T.

$$\eta = 20 \log \left[\frac{10RR_2A_d(S_c+S_d-\omega^2M_d)}{BL(R+R_1+R_2)S_c}\right] - T$$

When the measured values from paragraph 9.2 are inserted, η becomes

$$\eta = 20 \log \left[\frac{10X11 \quad 7X13 \quad 8(S_c + 30X10^6 - \omega^2 15.4)}{10 \quad 2X10^6 \quad S_c} \right] - T$$

$$\eta = 20 \log \left[\frac{1.58}{10^4} \left(1 + \frac{30X10^6 - 15.4\omega^2}{S_c} \right) \right] - T \qquad (11.1)$$

$$\eta = -76 + 20 \log \left[1 + \frac{30X16^6 - 15.4\omega^2}{S_c} \right] - T$$

Substituting equation (10 1) for S_c produces

$$\eta = -76 + 20 \log \left[1 + \frac{0\ 28 - 0.079h}{1.51X10^6 h} (30X10^6 - 15.4\omega^2) \right] - T \qquad (11.2)$$

This is the final equation for the calibration η in terms of the variables h and ω and the adjustable constant T.

12.0 CORRECTION FACTORS

The attenuator dial which adjusts T from 0 to +45 db is actually labeled from −80 to −125 db. Therefore, when the dial is adjusted to match e_A with e_H (see Fig 1), it will correctly read the hydrophone calibration η only if −80 is added to $-T$ Therefore,

$$\eta = -80 - T \qquad (12.1)$$

from equations (11.2) and (12.1).

If the dial is to read correctly, (11.2) and (12.1) must be equal, or

$$-76 + 20 \log \left[1 + \frac{0.028 - 0.079h}{1.51X10^6 h} (30X10^6 - 15.4\omega^2) \right] \stackrel{?}{=} -80 \qquad (12.2)$$

If, in a given condition, h and ω have values such that (12.2) is not an identity, a correction must be made. From equations (11.1) and (12.2), it is clear that if S_c were infinite, the variable part of (12.2) would drop out and (12.2) would reduce to

$$-76 \stackrel{?}{=} -80$$

A constant correction of −4 would then be required.

This constant correction factor is a result of the change of the system constants. A readjustment of the attenuator dial would completely eliminate it.

Since S_c is less than infinite, a variable correction is always required also. This "stiffness correction" is determined by

$$-76 + 20 \log \left[1 + \frac{0.028-0.079h}{1.51 \times 10^6 h} (30 \times 10^6 - 15.4\omega^2)\right] + \text{correction} = -80$$

or

$$\text{correction} = -4 - 20 \log \left[1 + \frac{0.028-0.079h}{1.51 \times 10^6 h} (30 \times 10^6 - 15.4\omega^2)\right] \qquad (12.3)$$

Correction charts or graphs can be prepared from equation (12.3) which will conveniently show the correction as a function of h and ω (or f where $f = \omega/2\pi$). The chart used as a result of this calibration is shown in Fig. 13.

13.0 CALIBRATION PRECISION

13.1 Theory

To find the over-all precision of the system as it is affected by the measurements made during the system calibration, the following conventional analysis was used:

(a) Starting with the estimated probable maximum error in each measured constant or variable, the error in η produced by each of these measurement errors was calculated.

(b) The square root of the sum of squares of these calculated errors was then found. This figure is the over-all precision of the system calibration.

(c) The precision was converted from absolute values to db measure. To illustrate,

let $\eta = f(x, y, z)$

and Δx = maximum error in x

Δy = maximum error in y

Δz = maximum error in z

Then $\epsilon_x = (\partial\eta/\partial x)\Delta x$ = error in η due to Δx

$\epsilon_y = (\partial\eta/\partial y)\Delta y$ = error in η due to Δy

$\epsilon_z = (\partial\eta/\partial z)\Delta z$ = error in η due to Δz.

The over-all precision will then be

$$E = \sqrt{\epsilon_x^2 + \epsilon_y^2 + \epsilon_z^2}$$

The theory requires all measurement errors to be independent of each other. In this calibration there has been some interdependence of the measurements; however, in the measurement of each constant, one or more totally independent methods have been used. With this in mind, the estimated maximum errors have been made large enough so that if only the totally independent measurements were used the precision of the system calibration would not be less than this analysis shows.

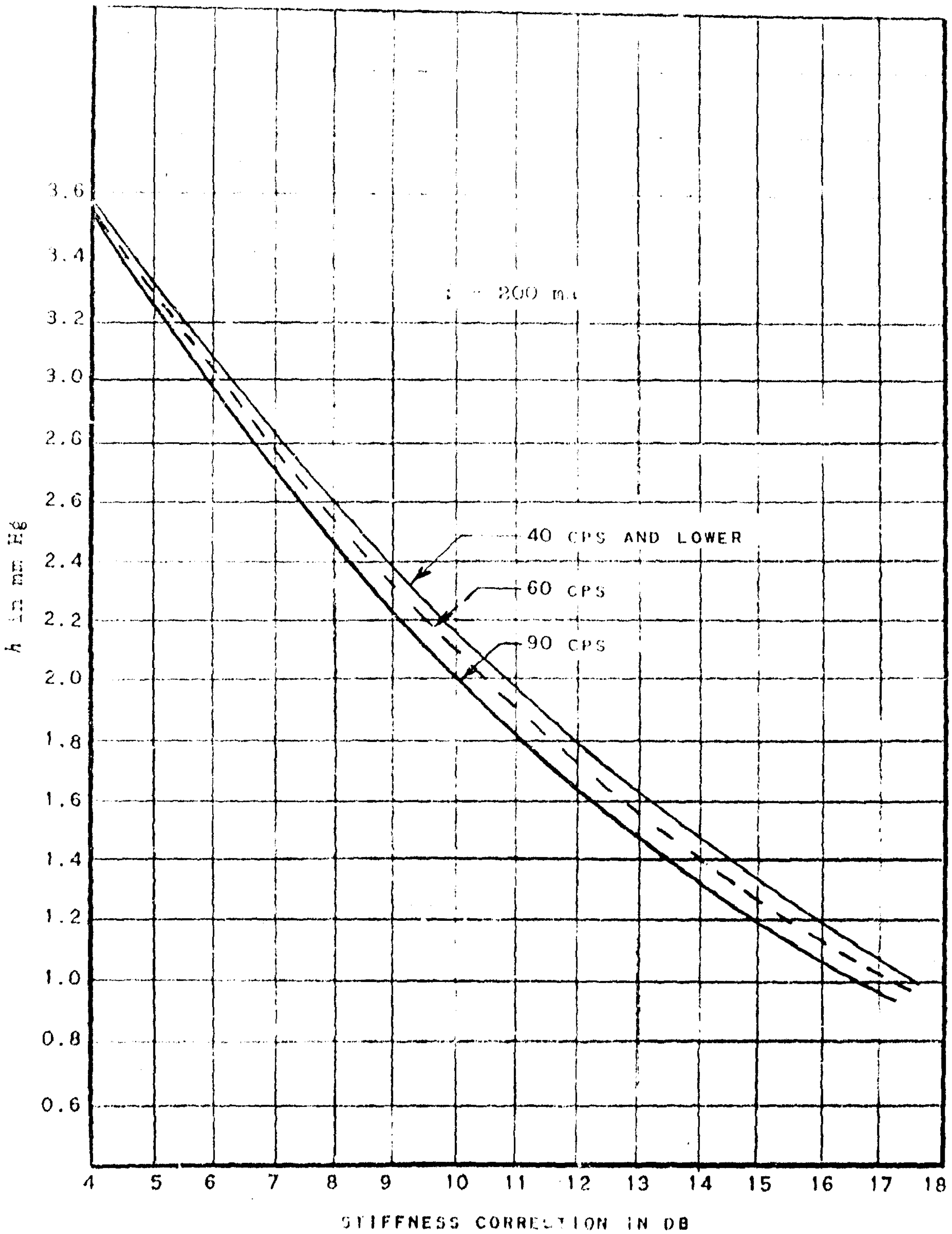


Fig. 13. Stiffness correction vs manometer difference

13.2 Analysis.

Equation (2.1) can be rearranged to

$$\eta = 20 \log \left[\frac{10RR_2}{R+R_1+R_2} \left(\frac{1}{BL} \right) (A_d) \left(1 + \frac{S_d}{S_c} - \frac{\omega^2 M_d}{S_c} \right) \right] - T$$

If we write

$$20 \log \eta' = \eta$$

$$20 \log T' = -T$$

$$\frac{10RR_2}{R+R_1+R_2} = K$$

then

$$\eta' = KT' \left[\frac{1}{BL} \right] (A_d) \left[1 + \frac{S_d}{S_c} - \frac{\omega^2 M_d}{S_c} \right] \tag{13.1}$$

From equations (2.2), (2.3), and (2.4)

$$\frac{1}{S_c} = \frac{(0.1)BLi}{2h\rho g A_d S_d} - \frac{1}{S_d} - \frac{A_m}{2\rho g A_d^{\,2}} \tag{13.2}$$

Substituting (13.2) into (13.1) and rearranging terms produces

$$\eta' = KT' \left[\frac{(0.1)i}{2h\rho g} - \frac{S_d A_m}{BL 2\rho g A_d} - \frac{\omega^2 M_d (0.1) i}{2h\rho g S_d} + \frac{\omega^2 M_d A_d}{S_d BL} + \frac{\omega^2 M_d A_m}{2\rho g A_d BL} \right] \tag{13.3}$$

Equation (13.3) gives η' as a function of twelve values:

(a) The measured constants, BL, S_d, M_d, A_m, A_d, and K, whose values and precisions are tabulated in paragraph 9.2.

(b) h, i, ω, and T' whose values and precisions are known from direct observation and use of the system equipment.

(c) ρ and g whose values and precisions are available from handbooks. Since ρ and g are used only together as a product, they can be combined into a single constant ρg.

The calculation requires an assumption of nominal values of all the variables and constants. These nominal values, together with probable maximum errors, are listed below. Note that T' is taken at 1. Using this value results in the computation of maximum errors in η'. ω is taken at two points, 0 and 200π, its minimum and maximum values.

	Nominal Value	Maximum Error	Units
BL	10^7	±0.2	gauss-cm
S_d	3×10^7	±1.0	dynes/cm
M_d	15	±0.2	grams
A_m	0.1	±0.002	cm^2
A_d	14	±0.2	cm^2
K	117	±3.5	ohms
T'	1	±0.03	---
h	0.2	±0.01	cm
i	0.2	±0.001	amp
ω	0 and 200π	$\pm 2\pi$	1/sec
ρg	14,000	±280	$gram/cm^2\text{-}sec^2$

13.3 Calculation

The partial differentiation of η' with respect to each constant or variable involves much tedious calculation which is superfluous to the purpose of this report. Consequently, only one variable, h, will be completely worked out, and only the results of the other calculations presented.

$$\epsilon_h = \frac{\partial\eta'}{\partial h}\Delta h = \frac{\partial}{\partial h}\left\{KT'\left(\frac{(0.01)i}{2h\rho g} - \frac{S_d A_m}{BL2\rho g A_d} - \frac{\omega^2 M_d(0.1)i}{2h\rho g S_d} + \frac{\omega^2 M_d A_d}{S_d BL} + \frac{\omega^2 M_d A_m}{2\rho g A_d BL}\right)\right\}(0.01)$$

$$\epsilon_h = \left[-\frac{KT'(0.1)i}{h^2 2\rho g} - 0 + \frac{KT'\omega^2 M_d(0.1)i}{h^2 2\rho g S_d} + 0 + 0\right](0.01)$$

$$\epsilon_h = \frac{-KT' i}{h^2 20\rho g}\left[1 - \frac{\omega^2 M_d}{S_d}\right](0.01)$$

Inserting the nominal values and taking $\omega = 200\pi$, we have

$$\epsilon_h = -\frac{(117)(0.2)}{(0.04)(20)(14,000)}\left[1 - \frac{6\times10^6}{3\times10^7}\right](0.01) = -1.67\times10^{-6}$$

and for $\omega = 0$

$$\epsilon_h = \frac{(117)(0.2)}{(0.04)(20)(14,000)}(1)(0.01) = -2.08\times10^{-6}$$

By similar calculation

			$(\omega = 200\pi)$	$(\omega = 0)$
ϵ_K	$= (\partial\eta'/\partial K)\Delta K$	$=$	$+9.34\times10^{-6}$	$+10.5\times10^{-6}$
$\epsilon_{T'}$	$= (\partial\eta'/\partial T')\Delta T'$	$=$	$+9.34\times10^{-6}$	$+10.5\times10^{-6}$
ϵ_i	$= (\partial\eta'/\partial i)\Delta i$	$=$	$+1.67\times10^{-6}$	$+2.08\times10^{-6}$
$\epsilon_{\rho g}$	$= (\partial\eta'/\partial[\rho g])\Delta(\rho g)$	$=$	-8.40×10^{-6}	-9.70×10^{-6}
ϵ_s	$= (\partial\eta'/\partial S_d)\Delta S_d$	$=$	$+1.14\times10^{-6}$	$+0.30\times10^{-6}$
ϵ_{A_m}	$= (\partial\eta'/\partial A_m)\Delta A_m$	$=$	-1.43×10^{-6}	-1.75×10^{-6}
ϵ_{BL}	$= (\partial\eta'/\partial BL)\Delta BL$	$=$	$+0.78\times10^{-6}$	$+1.79\times10^{-6}$
ϵ_{A_d}	$= (\partial\eta'/\partial A_d)\Delta A_d$	$=$	$+1.56\times10^{-6}$	$+1.28\times10^{-6}$
ϵ_ω	$= (\partial\eta'/\partial\omega)\Delta\omega$	$=$	-0.58×10^{-6}	0
ϵ_M	$= (\partial\eta'/\partial M_d)\Delta M_d$	$=$	$+6.00\times10^{-6}$	0

The over-all probable error, defined as $\Delta\eta'$ becomes

$$\Delta\eta' = \sqrt{\epsilon_K^2 + \epsilon_{T'}^2 + \epsilon_i^2 + \epsilon_{\rho g}^2 + \epsilon_s^2 + \epsilon_{A_m}^2 + \epsilon_{BL}^2 + \epsilon_{A_d}^2 + \epsilon_\omega^2 + \epsilon_M^2 + \epsilon_h^2}$$

At $\omega = 0$,

$$\Delta\eta' = \sqrt{10^{-12}(10.5^2+10.5^2+2.08^2+9.70^2+0.30^2+1.75^2+1.79^2+1.28^2+2.08^2)}$$

$$\Delta\eta' = \frac{\sqrt{331}}{10^6} = \frac{18.2}{10^6} \frac{\text{volts}}{\mu b}$$

At $\omega = 200\pi (f = 100$ cps)

$$\Delta\eta' = \sqrt{10^{-12}(9.34^2+9.34^2+1.67^2+8.40^2+1.14^2+1.43^2+0.78^2+1.56^2+0.58^2+6.0^2+1.67^2)}$$

$$\Delta\eta' = \frac{\sqrt{293}}{10^6} = \frac{17.1}{10^6} \frac{\text{volts}}{\mu b}$$

To convert these absolute values to "db" accuracy, the nominal magnitude of the hydrophone calibration must be found. We used $T' = 1$ which means the attenuator dial read a calibration of -80 db referred to one volt per microbar. From Fig. 13, and an h assumed to be 2.0 mm, a correction of 10.9 db at 0 cps, and 10.1 db at 100 cps are applied to -80 db. Thus,

η at 0 cps = -69.1 db

η at 100 cps = -69.9 db

or

$$\eta' = \frac{3.2}{10^4} \text{ for 0 cps } (\omega = 0)$$

$$\eta' = \frac{3.5}{10^4} \text{ for 100 cps } (\omega = 200\pi)$$

Then

$$\Delta\eta'/\eta' = (18.2/10^6)(10^4/3.2) = 0.0568 \text{ or } 5.7\% \text{ (at } f = 0)$$

$$\Delta\eta'/\eta' = (17.1/10^6)(10^4/3.5) = 0.0488 \text{ or } 4.9\% \text{ (at } f = 100)$$

When these percentages are converted to db, the results are as follows:

+5.7% = +0.48 db

-5.7% = -0.50 db

+4.9% = +0.40 db

-4.9% = -0.44 db

From the values above, the over-all precision of the system calibration can be estimated to be approximately ±0.5 db.

14.0 SUPPLEMENTARY NOTES ON RECENT CALIBRATIONS

Since the first calibration of the low-frequency system at the USRL, it has been necessary to replace the diaphragm assembly or magnet twice. Each such change has required a recalibration of the system. In general, the same analysis and techniques as are described in this report have been used in these subsequent calibrations, but several refinements in the techniques have been made and are discussed below.

14.1 *BL* Measurement

The use of a fluxmeter has simplified the measurement of the *BL* force factor to a large degree. A Rawson type 504 fluxmeter which measures a maximum of ±500,000 maxwells per turn of a search coil was used. The voice coil was used as the search coil, and moved through a measured displacement in the magnetic gap. This displacement was measured with a Starret dial indicator (No. 656-T3). The magnitude of this displacement was 0.002 inch, which is approximately the vibrational peak-to-peak amplitude of the diaphragm in normal use. Because the voice coil has a resistance larger than that specified for use with the Rawson fluxmeter, suitable corrections must be made.

The factor BL is calculated from the measured displacement and flux as follows:

$$B = \frac{\phi}{A} = \frac{\phi}{2\pi r \xi_d}$$

$$Bn = \frac{n\phi}{2\pi r \xi_d}$$

$$Bn2\pi r = \frac{n\phi}{\xi_d}$$

$$BL = \frac{n\phi}{\xi_d}$$

where ϕ = flux change, A = area change, n = coil turns, r = coil radius, ξ_d = coil displacement.

14.2 S_d Measurement

The use of an accurate dial indicator such as the Starret No. 653-T3 which can measure to an accuracy of ±0.00005 inch has lead to the exclusive use of the linear displacement methods discussed in paragraph 4.1.

14.3 M_d Measurement

The resonant frequency of the diaphragm changes with static pressure head or "at rest" location of the diaphragm and voice coil. This change has been found to be of sufficient magnitude to require resonant frequency measurements to be made under this normal static head, which is about 2 inches of mercury. This head is applied with a small vacuum or negative pressure in the inside of the driver.